Data Structures

Algorithms

Teacher : Wang Wei

1. Ellis Horowitz,etc., Fundamentals of Data Structures in C++
2. ,
3. http://inside.mines.edu/~dmehta/
4. ,

1

Why need algorithms

- To computer science
  - The concept of an algorithm is fundamental
- In developing large-scale computer systems
  - Algorithms
    - exist for many common problems
    - designing efficient algorithms plays a crucial role

2

Algorithm

- Definition
  - is a step-by-step procedure
  - a finite set of instructions to be executed in a certain order to get the desired output
    - if followed, accomplishes a particular task
- Algorithms are generally created independent of underlying languages

3

Characteristics

- **Input**
  - Zero or more quantities are externally supplied
- **Output**
  - At least one quantity is produced
- **Definiteness**
  - Each instructions is clear abs unambiguous
- **Finiteness**
  - If we trace out the instructions of an algorithm, then for all cases, the algorithm terminates after a finite number of steps
- **Effectiveness**
  - Every instruction must be basic enough to be carried out

Complexities

- Time Complexity of a program
  - is the amount of computer time it needs to run to completion
    - Running time or the execution time of operations of data structure must be as small as possible
- Space Complexity of a program
  - is the amount of memory it needs to run to completion
    - Memory usage of a data structure operation should be as little as possible

Performance Measurement for Time Complexity

- Posteriori testing
  - is concerned with obtaining the actual space and time requirements of a program

Example : Sequential Search

```
int seqsearch ( int a[ ], int n, int x )
{
//    a[0],…,a[n-1]                     x
//                                      -1
    int i = 0;
    while ( i < n && a[i] != x )
        i++;
    if ( i == n ) return   1;
    return i;
}
```

- **Measuring the computing time of a program**
  function ***time()*** **or** ***clock()***

Example:

```
double runTime;
double start, stop;
time(&start);
int k = seqsearch (a, n, x);
time(&stop);
runTime = stop     start;
cout << " RunTime : " << runTime << endl;
```

10

11

Performance analysis
for
Time Complexity

- **Priori estimates**
  - **to predict the growth in run time as the instance characteristics change**
  - **asymptotic notation**
    - **Big “oh” : O**

12

Asymptotic Notation

- $f(n) = O(g(n))$
  - **iff** (if and only if) **there exist positive constants c and n_0 such that $f(n) \le cg(n)$ for all n, $n \ge n_0$**
    - $g(n)$
      - **is an upper bound on the value**
      - **should be as small a function of n as one come up**

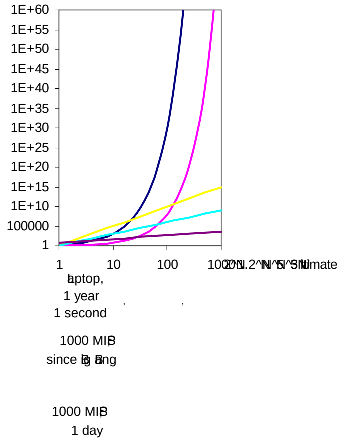
13

Theorem 1.2

- **if $f(n) = a_m n^m + \ldots + a_1 n + a_0$, then $f(n) = O(n^m)$**
  - **Proof :**

    $f(n) \le \sum_{i=0}^{m} |a_i| n^i \le n^m \sum_0^m |a_i| n^{i-m} \le n^m \sum_0^m |a_i|, n \ge 1$
  - **So, $f(n) = O(n^m)$**
  - When the complexity of an algorithm is actually, say, O(log n),
  - but we can only show that it is O(n) due to the limitation of our knowledge
  - it is OK to say so.
  - This is one benefit of O notation as upper bound.

14



Time complexity

- The time taken by a program ***P***
  $t(P) = c + t_P(n)$
  - ***c*** : constant
  - t_P : function $f_P(n)$
  - ***n*** : the number of the inputs and outputs

- $T(n) = O(f(n))$

16

- Compile time
- Run or execution time
  - *program step*
    - a syntactically or semantically meaningful segment of a program that has a run time
    - Run time is independent of n

17

- **Determine the number of steps : method 1**
  - Introduce a global variable ***count*** with initial value ***0***

```
int count=0;
float sum (float a[ ], int n)
{   float s = 0.0;        //count++
    count++;
    for (int i = 0; i < n; i++)   //count++ :  <init>;<expr1>
    {   count ++;
        s += a[i];         //count++
        count++;
    }
    count ++          //count++:  <expr1>;<expr2>
    count++;
    return s;    //count++  :  return
}
```

18

- **Determine the number of steps : method 2**
  - **build a *table***

s/e : steps per execution

program	s/e	frequency	steps
{	0	1	0
float s = 0.0;	1	1	1
for (**int** i=0; i<n; i++)	1	n+1	1n+1
s += a[i];	1	n	n
return s;	1	1	1
}	0	1	0
	total steps		2n+3

s/e : steps per execution

program	s/e	Frequency n=0/n>0	Steps n=0/n>0
{	0	1/1	0/0
if (n<=0)	1	1/1	1/1
return 0;	1	1/0	1/0
else			
return sum(a,n-1)+a[n-1]);	1+f(n-1)	0/1	0/1+f(n-1)
}	0	1/1	0/0
	total steps		2/ 2+f(n-1)

- $T(n, m) = T_1(n) + T_2(m)$
  $= O(\max(f(n), g(m)))$

```
x = 0;  y = 0;
```
$T_1(n) = O(1)$

```
for ( int k = 0; k < n; k ++ )
    x ++;
```
$T_2(n) = O(n)$

```
for ( int i = 0; i < n; i++ )
    for ( int j = 0; j < n; j++ )
        y ++;
```
$T_3(n) = O(n^2)$

$T(n) = T_1(n)+T_2(n)+T_3(n) = O(\max(1, n, n^2)) = O(n^2)$

```
void  bubbleSort (int a[ ],  int n )
{ //        a[ ]                 , n
     for (int i = 1; i <= n-1; i++)
     {    //n-1
         for (int j = n-1; j >= i; j--)   //n-i
            if (a[j-1] > a[j])
            {  int tmp = a[j-1];
               a[j-1] = a[j];
               a[j] = tmp;
            }     //
     }
}
```

22

· $T(n, m) = T_1(n) * T_2(m)$

$= O(f(n)*g(m))$

BubbleSort

n-1

n-i

$O(f(n)*g(n)) = O(n^2)$

$\because \sum_{i=1}^{n-1}(n-i) = \frac{n(n-1)}{2}$

23

Execution Time Cases

three cases

- **Worst Case**
  - This is the scenario where a particular data structure operation takes maximum time it can take.
  - If an operation's worst case time is *ƒ*(n) then this operation will not take more than *ƒ*(n) time where *ƒ*(n) represents function of n
- **Average Case**
  - This is the scenario depicting the average execution time of an operation of a data structure.
  - If an operation takes *ƒ*(n) time in execution, then m operations will take m*ƒ*(n) time
- **Best Case**
  - This is the scenario depicting the least possible execution time of an operation of a data structure.
  - If an operation takes *ƒ*(n) time in execution, then the actual operation may take time as the random number which would be maximum as *ƒ*(n)

24

Space complexity

- The space requirement of program ***P***

  S(P) = c + S_P(n)
  - ***c*** : constant
  - S_P : function $f_P(n)$
  - ***n*** : the number of the inputs and outputs

- **S(*n*) = O(*f*(*n*))**

25

- **Fixed part :** is independent of the number of the inputs and outputs
  - Space for the code
  - Constant
  - Simple variables
  - Fixed-size component variables

- **Variable part :** is dependent on the particular instance
  - component variables
  - Referenced variables
  - Recursion stack space

26

Example

```
//iterative function
 float  Sum (float *a, const int n)
 {  float s=0;
    for(int i=0;i<n;i++)
       s+=a[i];
   return  s;
 }
```

```
  //recursive function
  float  Rsum (float *a, const int n)
  {  if (n <=0)   return 0;
     else return (Rsum(a,n-1)+a[n-1]);
  }
```

27